Analysis of Pendulum Damper for Satellite Wobble Damping

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A theoretical investigation of the damping of the precession of an asymmetric spinning body by use of a pendulum damper shows that a pendulum, located arbitrarily in the satellite, with its pivot comprised of a viscoelastic material, can effectively reduce even the small wobble angles of the spinning body. The satellite has three distinct moments of inertia and the pendulum is free to rotate about two axes. Equations of motion of the coupled bodies were derived and an expression obtained for the wobble angle decay as a function of system parameters. Finally, a numerical example is presented to indicate the effectiveness of the pendulum damper.

Nomenclature

A, B, C	= mass moments of inertia about the body-fixed z, x, and y axes, respectively
c	= viscous damping coefficient at the pivot
G_2/G_1	= loss factor of the viscoelastic material
$\mathbf{h}$	= total angular momentum vector
$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	= unit vectors directed along the x, y, and z axes, respectively
k	= modulus of the elliptic function
K_T	= spring constant of torsional spring at the pivot
L	= pendulum rod length
m, W	= tip mass and weight of the pendulum
p	= precession frequency
$\mathbf{r}$	= radius vector from the origin of coordinates to the tip mass of the pendulum, resolved along the body axes
t	= time
T	= kinetic energy of the spinning body
x, y, z	= body-fixed orthogonal set of axes
x_0, y_0, z_0	= coordinates of the pendulum pivot point in the body-fixed axes system
X, Y, Z	= orthogonal set of axes fixed in inertial space
α, β	= rotations of the pendulum, measured in planes parallel to the z-y and x-z planes, respectively
θ, ϕ, ψ	= Euler angles defining the transformation from inertial to body-fixed coordinates
θ_m	= maximum value of the half-cone wobble angle in any precession cycle
θ_{m_0}	= initial value of the half-cone wobble angle
$\boldsymbol{\omega}$	= angular velocity vector
ω_n	= natural frequency of the pendulum
ω_x, ω_y, ω_z	= angular rates about the body-fixed x, y, and z axes, respectively

Introduction

SPIN stabilization is an accepted means of maintaining vehicle attitude, as a spinning body has a natural resistance to torques about axes other than the spin axis. However, a spin-stabilized body will generally exhibit conical wobble or free precession of its spin axis because of imperfections in the spacecraft ejection process and because of periodic attitude corrections and midcourse maneuvers. Most spin-stabilized vehicles will require that the spin axis be aligned with one of the principal axes within a minimum degree of tolerance. This is particularly true of photographic satellites, where any slight misalignment between spin and optical axes would impair the quality and interpretability of the photographs being taken. In the absence of an active system, a passive damper performs the function of dissipating the energy associated with wobble of the spin axis, eventually causing the spin axis to align itself in a desired attitude within preset limits.†

This paper presents an analysis of a passive damper designed to reduce the conical wobble or free precession of the spin axis of an asymmetric spinning body. The damper is pendulous, i.e., a concentrated mass at the end of a light, rigid arm. The pivot point of the pendulum is assumed to possess an omnidirectional torsional spring and viscous damper (see Fig. 1).

There have been many papers describing passive damping devices. Of particular interest are the papers by Taylor[2] and Carrier and Miles.[3] Taylor describes a single spring-mass system with a viscous damper in parallel with the spring providing the energy sink. Carrier and Miles study a damper that dissipates energy through viscous effects of a fluid moving around an annular ring.

One of the outstanding papers on pendulous dampers is by Haseltine.[4] He describes a system using a number of pendulums with pivot points and masses in a plane normal to the spin axis of the vehicle. The vehicle is symmetric, and each pendulum moves in a plane parallel to the plane of the transverse axes. The wobble is reduced by means of viscous dampers, attached in some manner to the pendulums and the body.

The pendulum damper described in this paper utilizes a viscoelastic material at its pivot to provide both a torsional spring and a viscous damper. The pendulum is located anywhere in the body, with its null position parallel to the desired direction of the spin axis. The particular advantage of a passive damper utilizing viscoelastic material as an energy sink (in a manner shown schematically in Fig. 2) over other mechanical-type passive dampers is the fact that it will not cease to function at low wobble angles because of coulomb friction (stiction). Hence, this type of damper would prove effective in cases where the initial wobble angle is small in absolute value, but still detrimental to the effective performance of the vehicle. Locating the pendulum parallel to the desired location of the spin axis and incorporating a spring in its pivot, it is possible to tune the damper to the precession frequency, assuring effectiveness of the damper for the particular inertial properties of the vehicle to which it is attached. In contrast, a pendulum in or parallel to the plane of the transverse axes of the vehicle will have a resonant frequency equal to or greater than the spin rate, depending on the location of its pivot point. It is not possible to tune this con-

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† It is assumed in this discussion that the body is spinning about an axis of maximum moment of inertia, which corresponds to a minimum energy condition. It is shown in Ref. 1 that a body spinning about any other body axis, in the presence of dissipative forces, will tend to align its spin axis with the axis of maximum moment of inertia. This would clearly represent an unstable condition for that body.

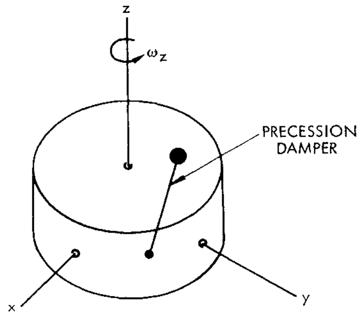

Fig. 1 Pendulum damper in spinning satellite.

figuration to the precession frequency. Finally, this paper considers the problem of spherical motion of the pendulum, and analyzes the general case of an asymmetric spinning body.

Motion of an Asymmetric Body

The vehicle has three distinct moments of inertia: A, B, and C. Assign the relationship $A > B > C$ to these quantities. Figure 3 references a set of body-fixed axes to an inertially fixed system through the use of the Euler angles θ, ϕ, and ψ. The total angular momentum vector is aligned along the Z axis of the inertial coordinate system. The angle θ is measured to the spin axis of the vehicle, corresponding to moment of inertia A. Moments of inertia B and C are about the x and y axes, respectively. The angular momentum and kinetic energy of the body are as follows:

$$h^2 = A^2\omega_z^2 + B^2\omega_x^2 + C^2\omega_y^2 \tag{1}$$

$$2T = A\omega_z^2 + B\omega_x^2 + C\omega_y^2 \tag{2}$$

The determination of ω_x, ω_y, and ω_z involves the use of elliptic functions (see Synge and Griffith,[6] p. 377). The cases of interest have relatively small wobble, and so with ω_z considerably greater than both ω_x and ω_y and $h^2 > 2BT$, the values of ω_x, ω_y and ω_z become

$$\begin{aligned} \omega_x &= \beta sn[p(t - t_0)] \\ \omega_y &= \gamma cn[p(t - t_0)] \\ \omega_z &= \alpha dn[p(t - t_0)] \end{aligned} \tag{3}$$

where

$$\begin{aligned} \alpha &= [(h^2 - 2CT)/A(A - C)]^{1/2} \\ \beta &= [(2AT - h^2)/B(A - B)]^{1/2} \\ \gamma &= -[(2AT - h^2)/C(A - C)]^{1/2} \\ k &= [(B - C)(2AT - h^2)/(A - B)(h^2 - 2CT)]^{1/2} \\ p &= [(A - B)(h^2 - 2CT)/ABC]^{1/2} \end{aligned} \tag{4}$$

where p is the precession frequency and k is the modulus of the elliptic function. The following relations between the angular velocities and Euler angles can be obtained from Fig. 3 by inspection:

$$\begin{aligned} \cos\theta &= A\omega_z/h \\ \tan\psi &= B\omega_x/C\omega_y \\ \dot{\phi} &= (\omega_x \sin\psi + \omega_y \cos\psi)/\sin\theta \end{aligned} \tag{5}$$

Equations (1–5) can be used to develop the following exact relationship involving the maximum value of θ, termed θ_m (see Ref. 5):

$$\begin{aligned} \alpha &= h\cos\theta_m/A(1 - k^2)^{1/2} \\ \beta &= (h/B)\sin\theta_m \\ \gamma &= -h\left[\frac{A - B}{BC(A - C)}\right]^{1/2}\sin\theta_m \end{aligned} \tag{6}$$

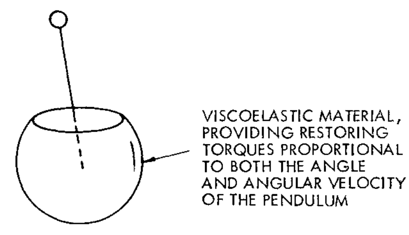

Fig. 2 Conceptual representation of pendulum and pivot.

$$k = \left[\frac{A(B - C)\sin^2\theta_m}{B(A - C) - C(A - B)\sin^2\theta_m}\right]^{1/2} \tag{7}$$

$$p = \left[\frac{h^2(A - B)(A - C)(1 - \sin^2\theta_m)}{A^2BC(1 - k^2)}\right]^{1/2} \tag{8}$$

$$T = \frac{h^2}{2A}\left[1 + \frac{(A - B)}{B}\sin^2\theta_m\right] \tag{9}$$

For θ on the order of 10° or less, the modulus k is very small, which allows the following approximation[6]:

$$sn\ u = \sin u \qquad cn\ u = \cos u \qquad dn\ u = 1 \tag{10}$$

Substituting Eqs. (6) and (10) into Eq. (3), and making small angle approximations, the following relationships are obtained:

$$\begin{aligned} \omega_x &= (h/B)\theta_m \sin[p(t - t_0)] \\ \omega_y &= -h\left[\frac{A - B}{BC(A - C)}\right]^{1/2}\theta_m \cos[p(t - t_0)] \\ \omega_z &= \frac{h}{A(1 - k^2)^{1/2}} \simeq \frac{h}{A} \end{aligned} \tag{11}$$

The last of Eqs. (11) indicates that the change in ω_z due to fluctuations in θ_m is so small that ω_z is, for all intent, constant.

Referring to Eq. (9), it is seen that the rate of change of kinetic energy is a function of the rate of change of moments of inertia and θ_m. It may be stated with a high degree of accuracy that the effect of the change in moments of inertia due to motion of the passive damper will be extremely small, considering the relative masses involved. Thus, holding these quantities constant, and neglecting the kinetic energy of internal motion, the rate of change of kinetic energy is

$$\dot{T} = \frac{h^2}{A}\frac{(A - B)}{B}\sin\theta_m\cos\theta_m\dot{\theta}_m$$

Making small angle assumptions

$$\dot{T} = h^2(A - B)\theta_m\dot{\theta}_m/AB \tag{12}$$

The rate of change of kinetic energy is equal to the nonrecoverable work done by the damping system averaged over a

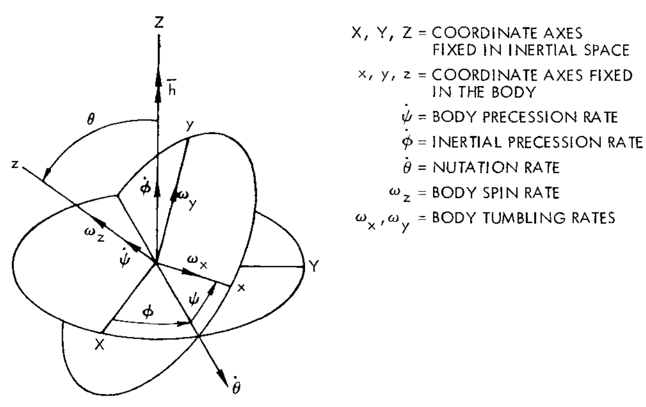

Fig. 3 Euler angle transformation.

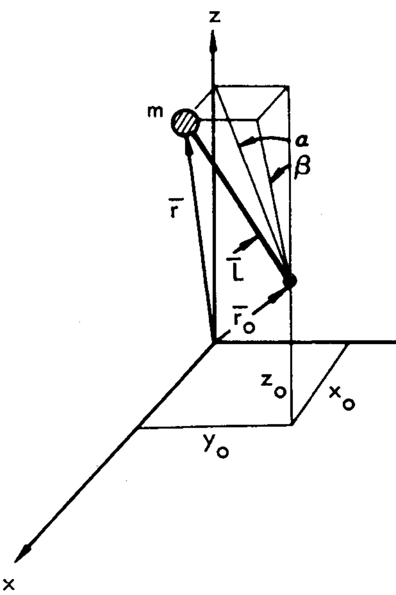

Fig. 4 Pendulum geometry.

period of time. Imagine a pendulum with a total rotation γ incorporating a viscous damper that supplies a torque N proportional to the angular velocity of the pendulum. Thus

$$N = c\dot{\gamma} \quad \text{and} \quad W = -\int_0^{\tau} c(\dot{\gamma})^2\, dt \tag{13}$$

Equating (12) with the average work done gives

$$\dot{T} = \frac{h^2}{A}\left(\frac{A-B}{B}\right)\theta_m\dot{\theta}_m = \frac{-c}{\tau}\int_0^{\tau}(\dot{\gamma})^2\, dt \tag{14}$$

Motion of the Pendulum

The pendulum is arbitrarily located in the vehicle with its pivot having coordinates (x_0, y_0, z_0) relative to the body-fixed x, y, z axis system. Since the pendulum mass will be small, its effect on the motion of the vehicle over any one precession cycle may be neglected (the cumulative energy-loss effect occurs over many cycles). The motion of the vehicle with damper is then given by Eq. (11).

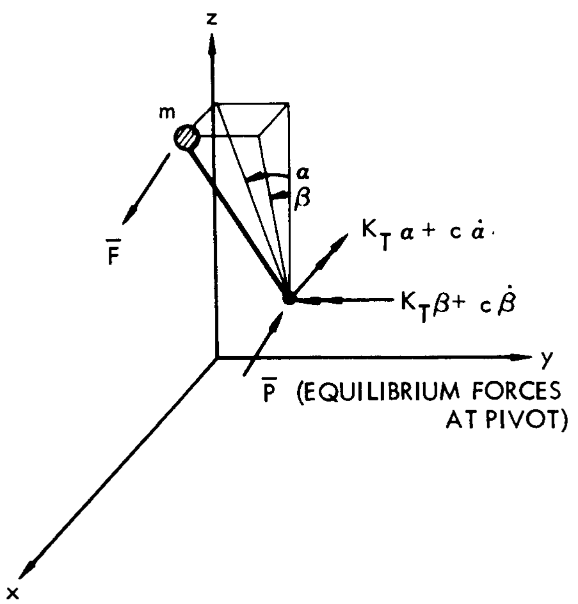

Fig. 5 Force and moment balance.

The rotations α and β are parallel to the z-y and x-z planes, respectively (see Fig. 4). The linear velocity of the mass m resolved along the body axis is

$$V = dr/dt = \dot{\mathbf{r}} + \boldsymbol{\omega}\times\mathbf{r} \tag{15}$$

The dot represents differentiation with respect to the body axes, whereas d/dt applies to the inertial coordinate system.

The angular velocity of the vehicle and the radius vector from the origin to the pendulum mass, both resolved along the body axes, may be expressed as follows:

$$\begin{aligned}\boldsymbol{\omega} &= \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\\ \mathbf{r} &= r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}\end{aligned} \tag{16}$$

i, **j**, and **k** are unit vectors directed along the x, y, and z axes, respectively.

The acceleration of the mass is

$$\mathbf{a} = d\mathbf{v}/dt = \dot{\mathbf{v}} + \boldsymbol{\omega}\times\mathbf{v} = \ddot{\mathbf{r}} + \dot{\boldsymbol{\omega}}\times\mathbf{r} + 2\boldsymbol{\omega}\times\dot{\mathbf{r}} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) \tag{17}$$

The "inertia force" due to this acceleration is

$$\mathbf{F} = -m\,(d\mathbf{v}/dt) \tag{18}$$

The equations can be expanded to obtain

$$\begin{aligned}F_x &= -m\,[\ddot{r}_x + \dot{\omega}_y r_z - \dot{\omega}_z r_y + 2(\omega_y\dot{r}_z - \omega_z\dot{r}_y) + \omega_x(\boldsymbol{\omega}\cdot\mathbf{r}) - r_x\omega^2]\\ F_y &= -m\,[\ddot{r}_y + \dot{\omega}_z r_x - \dot{\omega}_x r_z + 2(\omega_z\dot{r}_x - \omega_x\dot{r}_z) + \omega_y(\boldsymbol{\omega}\cdot\mathbf{r}) - r_y\omega^2]\\ F_z &= -m\,[\ddot{r}_z + \dot{\omega}_x r_y - \dot{\omega}_y r_x + 2(\omega_x\dot{r}_y - \omega_y\dot{r}_x) + \omega_z(\boldsymbol{\omega}\cdot\mathbf{r}) - r_z\omega^2]\end{aligned} \tag{19}$$

Applying D'Alembert's principle, these forces can be considered as opposing the motion of the pendulum, placing it in static equilibrium. Refer to Fig. 5, which infers the presence of a spring (spring constant K_T) and a viscous damper (viscous damping coefficient c) around each of the body axes. The equations of equilibrium are $\mathbf{L}\times\mathbf{F} + \mathbf{N} = 0$, where N is the torque. Thus

$$\begin{aligned}\Sigma M_x &= 0 \qquad L_yF_z - L_zF_y - (K_T\alpha + c\dot{\alpha}) = 0\\ \Sigma M_y &= 0 \qquad L_zF_x - L_xF_z - K_T\beta - c\dot{\beta} = 0\end{aligned} \tag{20}$$

Referring to Fig. 4,

$$\mathbf{r} = \mathbf{L} + \mathbf{r}_0 \tag{21}$$

$$\mathbf{L} = L\sin\beta\cos\alpha\mathbf{i} - L\cos\beta\sin\alpha\mathbf{j} + L\cos\beta\cos\alpha\mathbf{k} \tag{22}$$

$$\mathbf{r}_0 = x_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k} \tag{23}$$

so that

$$\begin{aligned}r_x &= L\sin\beta\cos\alpha + x_0\\ r_y &= -L\cos\beta\sin\alpha + y_0\\ r_z &= L\cos\beta\cos\alpha + z_0\end{aligned} \tag{24}$$

Substituting Eq. (24) into (18), assuming that the pendulum is restricted to small angular motion, and retaining only terms of first order, the following equations are obtained:

$$\left.\begin{aligned}F_x &= -mL\{\ddot{\beta} - \beta\omega_z^2 + 2\omega_z\dot{\alpha} + [(z_0/L) + 1](\dot{\omega}_y + \omega_x\omega_z) - (x_0/L)\omega_z^2\}\\ F_y &= -mL\{-\ddot{\alpha} + \alpha\omega_z^2 + 2\omega_z\dot{\beta} + [(z_0/L) + 1]\times(-\dot{\omega}_x + \omega_y\omega_z) - (y_0/L)\,\omega_z^2\}\\ F_z &= -m[y_0(\dot{\omega}_x + \omega_z\omega_y) + x_0(-\dot{\omega}_y + \omega_z\omega_x)]\end{aligned}\right\} \tag{25}$$

Substituting Eqs. (22, 24, and 25) into (20), and again making a first-order approximation, the equations of motion of the pendulum are obtained:

$$\ddot{\alpha} + \frac{c}{mL^2}\dot{\alpha} + \left(\frac{K_T}{mL^2} - \omega_z^2\right)\alpha - 2\omega_z\dot{\beta} = -\left[\left(\frac{z_0}{L}+1\right)(\dot{\omega}_x - \omega_y\omega_z)\right] - \frac{y_0}{L}\omega_z^2$$

$$\ddot{\beta} + \frac{c}{mL^2}\dot{\beta} + \left(\frac{K_T}{mL^2} - \omega_z^2\right)\beta + 2\omega_z\dot{\alpha} = -\left[\left(\frac{z_0}{L}+1\right)(\dot{\omega}_y + \omega_x\omega_z)\right] + \frac{x_0}{L}\omega_z^2 \qquad (26)$$

The terms on the right of Eq. (26) represent the forcing functions. From Eq. (11), the terms in brackets are

$$\dot{\omega}_x - \omega_y\omega_z = \left\{\frac{h^2}{A}\left[\frac{A-B}{BC(A-C)}\right]^{1/2} + \frac{hp}{B}\right\} \times \theta_m \cos[p(t-t_0)]$$

$$\dot{\omega}_y + \omega_x\omega_z = \left\{\frac{h^2}{AB} + hp\left[\frac{A-B}{BC(A-C)}\right]^{1/2}\right\} \times \theta_m \sin[p(t-t_0)] \qquad (27)$$

The steady-state solutions for Eqs. (26) are

$$\alpha_p = d\cos[p(t-t_0)] + s\sin[p(t-t_0)] - y_0\,\omega_z^2/GL$$

$$\beta_p = D\cos[p(t-t_0)] + S\sin[p(t-t_0)] + x_0\omega_z^2/GL \qquad (28)$$

$$\left.\begin{array}{ll} d = H(a\xi - b\eta) & s = H(a\eta + b\xi) \\ D = H(u\xi - v\eta) \text{ and } S = H(u\eta + v\xi) & \end{array}\right\} \qquad (29)$$

where

$$H \equiv [(z_0/L) + 1]\theta_m/(\xi^2 + \eta^2)$$

ξ, η, a, b, u, and v represent physical constants of the system and are as follows:

$$u = -Fp[(hM/A) + Rp]$$

$$v = -\left[2\omega_z p\left(\frac{hR}{A} + Mp\right) + (G - p^2)\left(\frac{hM}{A} + Rp\right)\right]$$

$$a = -\left[2\omega_z p\left(\frac{hM}{A} + Rp\right) + (G - p^2)\left(\frac{hR}{A} + Mp\right)\right]$$

$$b = Fp[(hR/A) + Mp]$$

$$\eta = 2Fp(G - p^2)$$

$$\xi = p^4 - p^2(F^2 + 2F + (2\omega_z)^2) + G^2$$

with

$$F = \frac{G}{p}\frac{G_2}{G_1} \qquad R = h\left[\frac{A-B}{BC(A-C)}\right]^{1/2}$$

$$G = (K_T/mL^2) - \omega_z^2 = \omega_n^2 \qquad M = h/B$$

Reduction of the Wobble Angle

Substitution of Eq. (28) into (15) with $(\dot{\gamma})^2 = (\dot{\alpha})^2 + (\dot{\beta})^2$ and integration over one cycle ($t = 2\pi/p$)

$$\theta_m\dot{\theta}_m = \frac{-AB}{h^2(A-B)}\frac{cp}{2\pi}\int_0^{2\pi/p}[(\dot{\alpha})^2 + (\dot{\beta})^2]\,dt \qquad (30)$$

results in the following expression

$$\theta_m\dot{\theta}_m = \frac{-ABcp^2}{2h^2(A-B)}(s^2 + d^2 + S^2 + D^2) \qquad (31)$$

Substituting from Eq. (29)

$$\frac{\dot{\theta}_m}{\theta_m} = \frac{-cp^2AB[(z_0/L)+1)^2}{2h^2(A-B)} \times \left[\frac{(a\xi - b\eta)^2 + (a\eta + b\xi)^2 + (u\xi - v\eta)^2 + (u\eta + v\xi)^2}{(\xi^2+\eta^2)^2}\right] \qquad (32)$$

The term in brackets relates to the physical constants of the satellite, the characteristics of the viscous damper, and the natural frequency of the pendulum $K_T/mL^2 - \omega_z^2$.

In any application of this theory, it will be the usual case that a particular damping rate will be required. The damper will be designed to satisfy this condition. Replacing the term in brackets in Eq. (32) by U, the entire right side of the equation can be expressed as

$$\frac{cp^2AB[(z_0/L)+1]^2}{2h^2(A-B)}U = \frac{1}{n} \qquad (33)$$

where n is dependent upon the physical characteristics of the satellite (which are usually known), and the characteristics of the damper, which are to be designed to match these requirements. Thus

$$\int_{\theta_{m0}}^{\theta_m}\frac{d\theta}{\theta} = -\frac{1}{n}\int_0^t dt$$

and

$$\theta_m = \theta_{m0}\,e^{-t/n} \qquad (34)$$

Damping Concepts

A viscoelastic material has an elastic modulus defined as

$$EM = G_1 + iG_2 \qquad (35)$$

G_1 is in phase with the deflection and hence represents that portion of the motion which is recoverable. G_2 is in phase with the velocity, and represents the nonrecoverable motion of the material. It is convenient to relate a viscous damping coefficient c to the properties of the viscoelastic material, i.e., G_1 and G_2, when both are used in a pendulus system. Assume that the system has parameters as follows:

m = tip mass (mass of rod is neglected)
J = mass moment of inertia about the pivot
ω_n = natural frequency

Assume further that the pendulum is subjected to a harmonic forcing function with frequency ω. Comparing the equations of motion, term by term, of two such systems, one with a torsional spring and dashpot with viscous damping coefficient c at the pivot, the other using only a viscoelastic material at the pivot, it is simple to show by direct analogy that, for the two systems to exhibit similar energy absorption characteristics, the following relation must hold:

$$c = (J\omega_n^2/\omega)(G_2/G_1) \qquad (36)$$

Numerical Illustration

As an illustration of the effectiveness of this type of passive damper for certain applications, an example is presented, with physical constants and system parameters corresponding to a particular vehicle studied by the author with a view toward utilization of a pendulum damper for the reduction of wobble. The following values are used: $A = 67.1$, $B = 63.3$, and $C = 25.0$ slug-ft^2; $p = 0.499$ and $\omega_z = 1.5$ rad/sec; $x_0 = y_0 = 0$ and $z_0 = 1.25$ ft; $h = A\omega_z = 105.3$ slug-ft^2/sec.; $\theta_{m0} = 1° = 0.0175$ and $\theta_m = \frac{1}{2}° = 0.00873$ rad. The loss factor $\eta = G_2/G_1$ of a viscoelastic material is a function of the material.

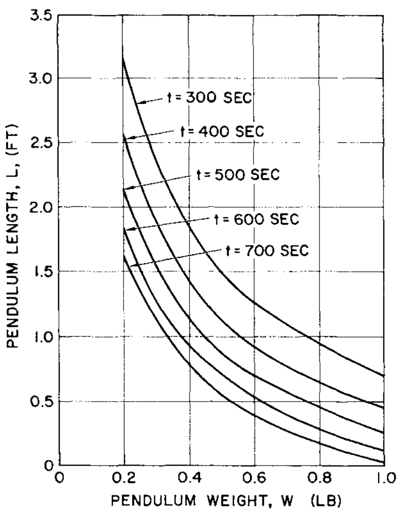

Fig. 6 Pendulum length vs weight (for several damping times).

It can be as low as 0.2. This value has been selected, hence adding a degree of conservatism to the analysis.

The analysis described in the preceding paragraphs is valid for small angular motion of the pendulum. If the initial half-cone wobble angle θ_{m0} is in the order of a few degrees, the pendulum frequency can be essentially tuned to the precession frequency without invalidating this assumption. The closeness of these two frequencies is therefore dependent on the damping rate desired and the loss factor of the viscoelastic material. For the example presented, a loss factor of 0.2 has been assumed. Iterating through Eqs. (29, 30, and 34) with the constants defined previously, it can be shown that, for $\omega_n = 5p$, a satisfactory damping rate can be achieved without compromising the assumptions of the analysis. Hence, this value, $\omega_n = 5p$, is chosen as a parameter of the pendulum. Substituting the necessary parameters into Eqs. (29) and (33), one obtains $U = 0.626$, and $1/n = 0.0196m\ (z_0 + L)^2$.

Substituting this into Eq. (34), one can obtain the following relation for the time to reduce the wobble angle to one-half its initial value:

$$t = 35.4/m(z_0 + L)^2 = 1140/W(L + 1.25)^2 \qquad (37)$$

Equation (37) is plotted in Fig. 6, which is then the time to damp from $\theta = 1°$ to $\theta = \frac{1}{2}°$ (or to halve the initial wobble angle), for various values of W and L.

It has been shown that

$$(K_t/mL^2) - \omega_z^2 = 25\ p^2$$

Solving for K_t, and substituting for m from Eq. (37), the following expression for K_t is obtained:

$$K_t = 35.4\ L^2(25\ p^2 + \omega_z^2)/t(1.25 + L)^2 \qquad (38)$$

This relation is plotted in Fig. 7 for several values of t.

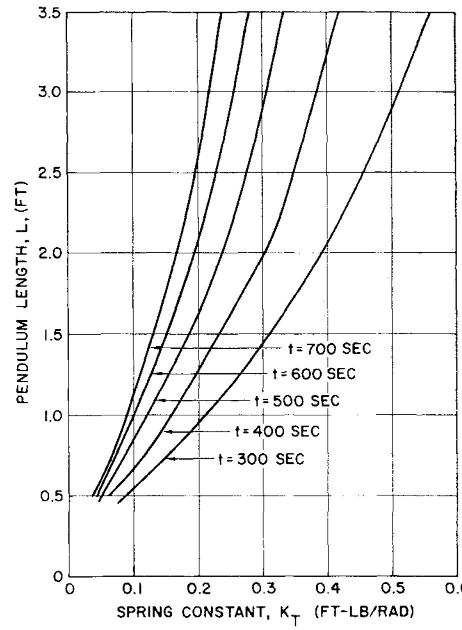

Fig. 7 Pendulum length vs spring constant (for several damping times).

Conclusions

It has been shown that the rate of decay of the wobble angle is exponential and can be simply expressed in terms of the initial half-cone wobble angle, the physical constants of the satellite, and the properties of the damper. Using fairly conservative estimates for the damper characteristics, and tuning the pendulum frequency to be slightly off the precession frequency, a satisfactory damping rate is achieved for a low-weight damping system. Finally, an analogy is presented showing the relation between the theoretical viscous damping factor and the loss factor of the viscoelastic material providing the energy sink and restoring torque for the system. This facilitates the physical development of a pendulum damper designed to perform in accordance with the concepts presented in this paper.

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